OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4764

Mechanics 4

Wednesday **21 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

Section A (24 marks)

1 A spherical raindrop falls through a stationary cloud. Water condenses on the raindrop and it gains mass at a rate proportional to its surface area. At time t the radius of the raindrop is r. Initially the raindrop is at rest and $r = r_0$. The density of the water is ρ.

(i) Show that $\frac{\mathrm{d}r}{\mathrm{d}t} = k$, where k is a constant. Hence find the mass of the raindrop in terms of r_0, ρ, k and t. [6]

(ii) Assuming that air resistance is negligible, find the velocity of the raindrop in terms of r_0, k and t. [6]

2 A rigid circular hoop of radius a is fixed in a vertical plane. At the highest point of the hoop there is a small smooth pulley, P. A light inextensible string AB of length $\frac{5}{2}a$ is passed over the pulley.

A particle of mass m is attached to the string at B. PB is vertical and angle APB $= \theta$. A small smooth ring of mass m is threaded onto the hoop and attached to the string at A. This situation is shown in Fig. 2.

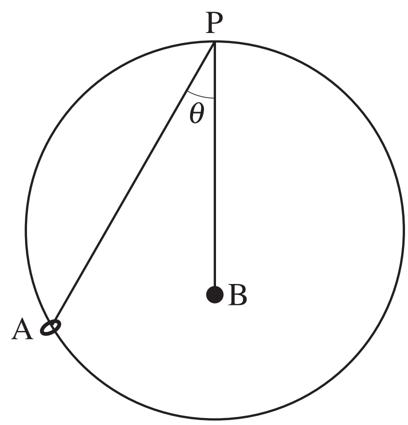

Fig. 2

(i) Show that PB $= \frac{5}{2}a - 2a\cos\theta$ and hence show that the potential energy of the system relative to P is $V = -mga(2\cos^2\theta - 2\cos\theta + \frac{5}{2})$. [4]

(ii) Hence find the positions of equilibrium and investigate their stability. [8]

Section B (48 marks)

3 An aeroplane is taking off from a runway. It starts from rest. The resultant force in the direction of motion has power, P watts, modelled by

$$P = 0.0004m(10\,000v + v^3),$$

where m kg is the mass of the aeroplane and v m s^{-1} is the velocity at time t seconds. The displacement of the aeroplane from its starting point is x m.

To take off successfully the aeroplane must reach a speed of 80 m s^{-1} before it has travelled 900 m.

(i) Formulate and solve a differential equation for v in terms of x. Hence show that the aeroplane takes off successfully. [8]

(ii) Formulate a differential equation for v in terms of t. Solve the differential equation to show that $v = 100\tan(0.04t)$. What feature of this result casts doubt on the validity of the model? [7]

(iii) In fact the model is only valid for $0 \leqslant t \leqslant 11$, after which the power remains constant at the value attained at $t = 11$. Will the aeroplane take off successfully? [9]

[Question 4 is printed overleaf.]

4 A flagpole AB of length $2a$ is modelled as a thin rigid rod of variable mass per unit length given by

$$\rho = \frac{M}{8a^2}(5a - x),$$

where x is the distance from A and M is the mass of the flagpole.

(i) Show that the moment of inertia of the flagpole about an axis through A and perpendicular to the flagpole is $\frac{7}{6}Ma^2$. Show also that the centre of mass of the flagpole is at a distance $\frac{11}{12}a$ from A. [8]

The flagpole is hinged to a wall at A and can rotate freely in a vertical plane. A light inextensible rope of length $2\sqrt{2}a$ is attached to the end B and the other end is attached to a point on the wall a distance $2a$ vertically above A, as shown in Fig. 4. The flagpole is initially at rest when lying vertically against the wall, and then is displaced slightly so that it falls to a horizontal position, at which point the rope becomes taut and the flagpole comes to rest.

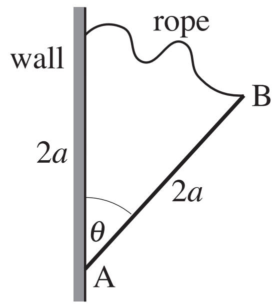

Fig. 4

(ii) Find an expression for the angular velocity of the flagpole when it has turned through an angle θ. [4]

(iii) Show that the vertical component of the impulse in the rope when it becomes taut is $\frac{1}{12}M\sqrt{77ag}$. Hence write down the horizontal component. [5]

(iv) Find the horizontal and vertical components of the impulse that the hinge exerts on the flagpole when the rope becomes taut. Hence find the angle that this impulse makes with the horizontal. [7]

ADVANCED GCE UNIT

4764/01

MATHEMATICS (MEI)

Mechanics 4

FRIDAY 22 JUNE 2007

Morning

Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
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ADVICE TO CANDIDATES

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[Turn over

Section A (24 marks)

1 A light elastic string has one end fixed to a vertical pole at A. The string passes round a smooth horizontal peg, P, at a distance a from the pole and has a smooth ring of mass m attached at its other end B. The ring is threaded onto the pole below A. The ring is at a distance y below the horizontal level of the peg. This situation is shown in Fig. 1.

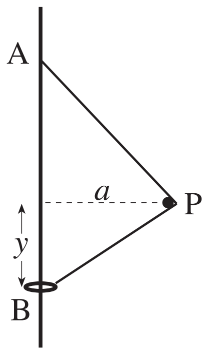

Fig. 1

The string has stiffness k and natural length equal to the distance AP.

(i) Express the extension of the string in terms of y and a. Hence find the potential energy of the system relative to the level of P. [5]

(ii) Use the potential energy to find the equilibrium position of the system, and show that it is stable. [5]

(iii) Calculate the normal reaction exerted by the pole on the ring in the equilibrium position. [2]

2 A railway truck of mass m_0 travels along a horizontal track. There is no driving force and the resistances to motion are negligible. The truck is being filled with coal which falls vertically into it at a mass rate k. The process starts as the truck passes a point O with speed u. After time t, the truck has velocity v and the displacement from O is x.

(i) Show that $v = \dfrac{m_0 u}{m_0 + kt}$ and find x in terms of m_0, u, k and t. [9]

(ii) Find the distance that the truck has travelled when its speed has been halved. [3]

Section B (48 marks)

3 **(i)** Show, by integration, that the moment of inertia of a uniform rod of mass m and length $2a$ about an axis through its centre and perpendicular to the rod is $\frac{1}{3}ma^2$. [6]

A pendulum of length 1 m is made by attaching a uniform sphere of mass 2 kg and radius 0.1 m to the end of a uniform rod AB of mass 1.2 kg and length 0.8 m, as shown in Fig. 3. The centre of the sphere is collinear with A and B.

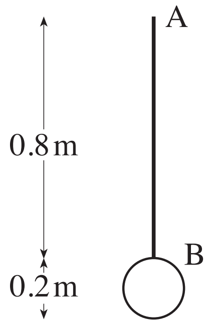

Fig. 3

(ii) Find the moment of inertia of the pendulum about an axis through A perpendicular to the rod. [7]

The pendulum can swing freely in a vertical plane about a fixed horizontal axis through A.

(iii) The pendulum is held with AB at an angle α to the downward vertical and released from rest. At time t, AB is at an angle θ to the vertical. Find an expression for $\dot{\theta}^2$ in terms of θ and α. [6]

(iv) Hence, or otherwise, show that, provided that α is small, the pendulum performs simple harmonic motion. Calculate the period. [5]

4 A particle of mass 2 kg starts from rest at a point O and moves in a horizontal line with velocity $v\,\mathrm{m\,s^{-1}}$ under the action of a force F N, where $F = 2 - 8v^2$. The displacement of the particle from O at time t seconds is x m.

(i) Formulate and solve a differential equation to show that $v^2 = \frac{1}{4}(1 - e^{-8x})$. [7]

(ii) Hence express F in terms of x and find, by integration, the work done in the first 2 m of the motion. [6]

(iii) Formulate and solve a differential equation to show that $v = \frac{1}{2}\left(\frac{1-e^{-4t}}{1+e^{-4t}}\right)$. [7]

(iv) Calculate v when $t = 1$ and when $t = 2$, giving your answers to four significant figures. Hence find the impulse of the force F over the interval $1 \leqslant t \leqslant 2$. [4]

RECOGNISING ACHIEVEMENT

ADVANCED GCE **4764/01**

MATHEMATICS (MEI)

Mechanics 4

WEDNESDAY 18 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
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This document consists of **4** printed pages.

Section A (24 marks)

1 A rocket in deep space starts from rest and moves in a straight line. The initial mass of the rocket is m_0 and the propulsion system ejects matter at a constant mass rate k with constant speed u relative to the rocket. At time t the speed of the rocket is v.

(i) Show that while mass is being ejected from the rocket, $(m_0 - kt)\dfrac{\mathrm{d}v}{\mathrm{d}t} = uk$. [5]

(ii) Hence find an expression for v in terms of t. [4]

(iii) Find the speed of the rocket when its mass is $\frac{1}{3}m_0$. [3]

2 A car of mass m kg starts from rest at a point O and moves along a straight horizontal road. The resultant force in the direction of motion has power P watts, given by $P = m(k^2 - v^2)$, where $v\,\mathrm{m\,s^{-1}}$ is the velocity of the car and k is a positive constant. The displacement from O in the direction of motion is x m.

(i) Show that $\left(\dfrac{k^2}{k^2 - v^2} - 1\right)\dfrac{\mathrm{d}v}{\mathrm{d}x} = 1$, and hence find x in terms of v and k. [9]

(ii) How far does the car travel before reaching 90% of its terminal velocity? [3]

Section B (48 marks)

3 A circular disc of radius a m has mass per unit area $\rho\,\mathrm{kg\,m^{-2}}$ given by $\rho = k(a + r)$, where r m is the distance from the centre and k is a positive constant. The disc can rotate freely about an axis perpendicular to it and through its centre.

(i) Show that the mass, M kg, of the disc is given by $M = \frac{5}{3}k\pi a^3$, and show that the moment of inertia, $I\,\mathrm{kg\,m^2}$, about this axis is given by $I = \frac{27}{50}Ma^2$. [9]

For the rest of this question, take $M = 64$ and $a = 0.625$.

The disc is at rest when it is given a tangential impulsive blow of 50 N s at a point on its circumference.

(ii) Find the angular speed of the disc. [4]

The disc is then accelerated by a constant couple reaching an angular speed of $30\,\mathrm{rad\,s^{-1}}$ in 20 seconds.

(iii) Calculate the magnitude of this couple. [3]

When the angular speed is $30\,\mathrm{rad\,s^{-1}}$, the couple is removed and brakes are applied to bring the disc to rest. The effect of the brakes is modelled by a resistive couple of $3\dot{\theta}$ N m, where $\dot{\theta}$ is the angular speed of the disc in $\mathrm{rad\,s^{-1}}$.

(iv) Formulate a differential equation for $\dot{\theta}$ and hence find $\dot{\theta}$ in terms of t, the time in seconds from when the brakes are first applied. [7]

(v) By reference to your expression for $\dot{\theta}$, give a brief criticism of this model for the effect of the brakes. [1]

4 A uniform smooth pulley can rotate freely about its axis, which is fixed and horizontal. A light elastic string AB is attached to the pulley at the end B. The end A is attached to a fixed point such that the string is vertical and is initially at its natural length with B at the same horizontal level as the axis. In this position a particle P is attached to the highest point of the pulley. This initial position is shown in Fig. 4.1.

The radius of the pulley is a, the mass of P is m and the stiffness of the string AB is $\frac{mg}{10a}$.

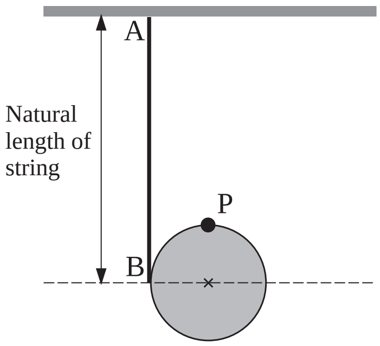

Fig. 4.1

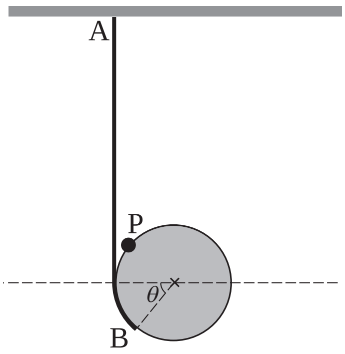

Fig. 4.2

(i) Fig. 4.2 shows the system with the pulley rotated through an angle θ and the string stretched. Write down the extension of the string and hence find the potential energy, V, of the system in this position. Show that $\frac{dV}{d\theta} = mga\left(\frac{1}{10}\theta - \sin\theta\right)$. [6]

(ii) Hence deduce that the system has a position of unstable equilibrium at $\theta = 0$. [6]

(iii) Explain how your expression for V relies on smooth contact between the string and the pulley. [2]

Fig. 4.3 shows the graph of the function $f(\theta) = \frac{1}{10}\theta - \sin\theta$.

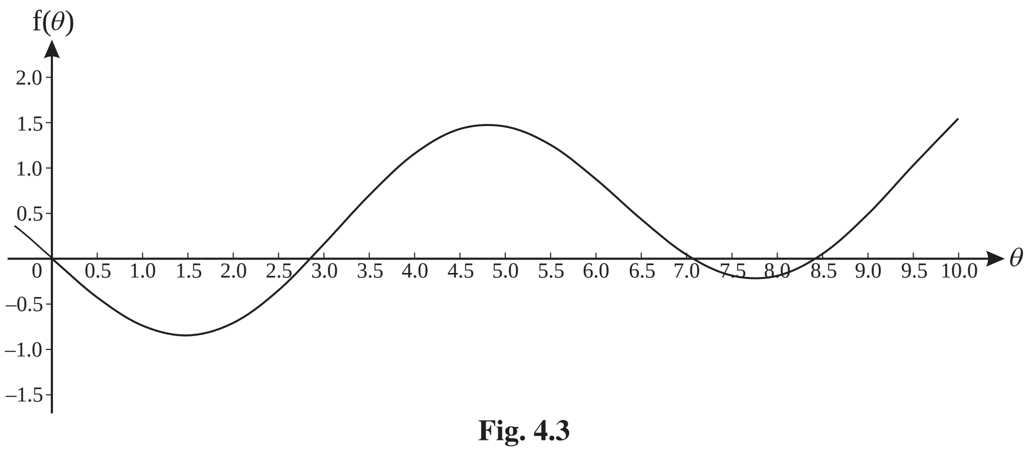

Fig. 4.3

(iv) Use the graph to give rough estimates of three further values of θ (other than $\theta = 0$) which give positions of equilibrium. In each case, state with reasons whether the equilibrium is stable or unstable. [6]

(v) Show on a sketch the physical situation corresponding to the least value of θ you identified in part **(iv)**. On your sketch, mark clearly the positions of P and B. [2]

(vi) The equation $f(\theta) = 0$ has another root at $\theta \approx -2.9$. Explain, with justification, whether this necessarily gives a position of equilibrium. [2]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4764**

Mechanics 4

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Thursday 11 June 2009
Morning

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

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Section A (24 marks)

1 A raindrop increases in mass as it falls vertically from rest through a stationary cloud. At time t s the velocity of the raindrop is $v\,\mathrm{m\,s^{-1}}$ and its mass is m kg. The rate at which the mass increases is modelled as $\dfrac{mg}{2(v+1)}\,\mathrm{kg\,s^{-1}}$. Resistances to motion are neglected.

(i) Write down the equation of motion of the raindrop. Hence show that

$$\left(1 - \frac{1}{v+2}\right)\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{2}g.$$ **[5]**

(ii) Solve this differential equation to find an expression for t in terms of v. Calculate the time it takes for the velocity of the raindrop to reach $10\,\mathrm{m\,s^{-1}}$. **[5]**

(iii) Describe, with reasons, what happens to the acceleration of the raindrop for large values of t. **[2]**

2 A uniform rigid rod AB of mass m and length $4a$ is freely hinged at the end A to a horizontal rail. The end B is attached to a light elastic string BC of modulus $\frac{1}{2}mg$ and natural length a. The end C of the string is attached to a ring which is small, light and smooth. The ring can slide along the rail and is always vertically above B. The angle that AB makes below the rail is θ. The system is shown in Fig. 2.

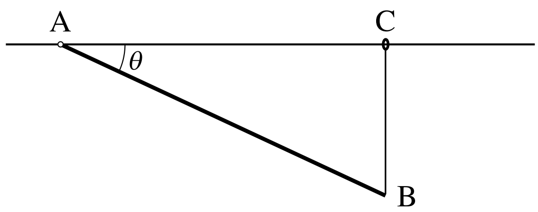

Fig. 2

(i) Find the potential energy, V, of the system when the string is stretched and show that

$$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 4mga\cos\theta(2\sin\theta - 1).$$ **[5]**

(ii) Hence find any positions of equilibrium of the system and investigate their stability. **[7]**

Section B (48 marks)

3 A uniform circular disc has mass M and radius a. The centre of the disc is at point C.

(i) Show by integration that the moment of inertia of the disc about an axis through C and perpendicular to the disc is $\frac{1}{2}Ma^2$. **[6]**

The point A on the disc is at a distance $\frac{1}{10}a$ from its centre.

(ii) Show that the moment of inertia of the disc about an axis through A and perpendicular to the disc is $0.51Ma^2$. **[2]**

The disc can rotate freely in a vertical plane about an axis through A that is horizontal and perpendicular to the disc. The disc is held slightly displaced from its stable equilibrium position and is released from rest. In the motion that follows, the angle that AC makes with the downward vertical is θ.

(iii) Write down the equation of motion for the disc. Assuming θ remains sufficiently small throughout the motion, show that the disc performs approximate simple harmonic motion and determine the period of the motion. **[6]**

A particle of mass m is attached at a point P on the circumference of the disc, so that the centre of mass of the system is now at A.

(iv) Sketch the position of P in relation to A and C. Find m in terms of M and show that the moment of inertia of the system about the axis through A and perpendicular to the disc is $0.6Ma^2$. **[5]**

The system now rotates at a constant angular speed ω about the axis through A.

(v) Find the kinetic energy of the system. Hence find the magnitude of the constant resistive couple needed to bring the system to rest in n revolutions. **[5]**

4 A parachutist of mass 90 kg falls vertically from rest. The forces acting on her are her weight and resistance to motion R N. At time t s the velocity of the parachutist is v m s^{-1} and the distance she has fallen is x m.

While the parachutist is in free-fall (i.e. before the parachute is opened), the resistance is modelled as $R = kv^2$, where k is a constant. The terminal velocity of the parachutist in free-fall is 60 m s^{-1}.

(i) Show that $k = \dfrac{g}{40}$. **[2]**

(ii) Show that $v^2 = 3600\left(1 - e^{-\frac{gx}{1800}}\right)$. **[7]**

When she has fallen 1800 m, she opens her parachute.

(iii) Calculate, by integration, the work done against the resistance before she opens her parachute. Verify that this is equal to the loss in mechanical energy of the parachutist. **[7]**

As the parachute opens, the resistance instantly changes and is now modelled as $R = 90v$.

(iv) Calculate her velocity just before opening the parachute, correct to four decimal places. **[1]**

(v) Formulate and solve a differential equation to calculate the time it takes after opening the parachute to reduce her velocity to 10 m s^{-1}. **[7]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4764**

Mechanics 4

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Tuesday 15 June 2010
Morning

Duration: 1 hour 30 minutes

* 4 7 6 4 *

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Section A (24 marks)

1 At time t a rocket has mass m and is moving vertically upwards with velocity v. The propulsion system ejects matter at a constant speed u relative to the rocket. The only additional force acting on the rocket is its weight.

(i) Derive the differential equation $m\dfrac{dv}{dt} + u\dfrac{dm}{dt} = -mg$. **[4]**

The rocket has initial mass m_0 of which 75% is fuel. It is launched from rest. Matter is ejected at a constant mass rate k.

(ii) Assuming that the acceleration due to gravity is constant, find the speed of the rocket at the instant when all the fuel is burnt. **[8]**

2 A particle of mass m kg moves horizontally in a straight line with speed $v\,\text{m s}^{-1}$ at time t s. The total resistance force on the particle is of magnitude $mkv^{\frac{3}{2}}$ N where k is a positive constant. There are no other horizontal forces present. Initially $v = 25$ and the particle is at a point O.

(i) Show that $v = 4\left(kt + \frac{2}{5}\right)^{-2}$. **[7]**

(ii) Find the displacement from O of the particle at time t. **[3]**

(iii) Describe the motion of the particle as t increases. **[2]**

Section B (48 marks)

3 A uniform rod AB of mass m and length $4a$ is hinged at a fixed point C, where AC = a, and can rotate freely in a vertical plane. A light elastic string of natural length $2a$ and modulus λ is attached at one end to B and at the other end to a small light ring which slides on a fixed smooth horizontal rail which is in the same vertical plane as the rod. The rail is a vertical distance $2a$ above C. The string is always vertical. This system is shown in Fig. 3 with the rod inclined at θ to the horizontal.

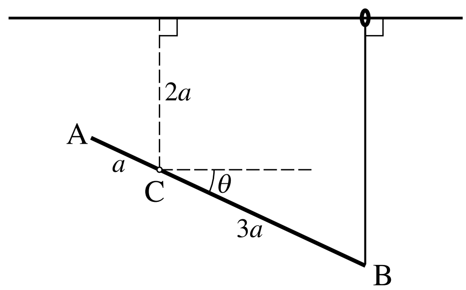

Fig. 3

(i) Find an expression for V, the potential energy of the system relative to C, and show that $\dfrac{dV}{d\theta} = a\cos\theta\left(\frac{9}{2}\lambda\sin\theta - mg\right)$. **[6]**

(ii) Determine the positions of equilibrium and the nature of their stability in the cases

(A) $\lambda > \frac{2}{9}mg$, **[10]**

(B) $\lambda < \frac{2}{9}mg$, **[4]**

(C) $\lambda = \frac{2}{9}mg$. **[4]**

4 Fig. 4.1 shows a uniform cone of mass M, base radius a and height $2a$.

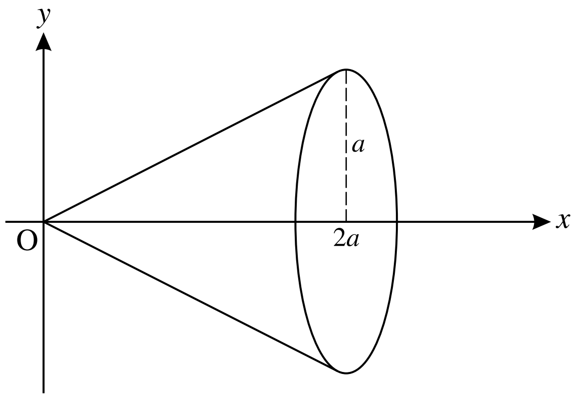

Fig. 4.1

(i) Show, by integration, that the moment of inertia of the cone about its axis of symmetry is $\frac{3}{10}Ma^2$. [You may assume the standard formula for the moment of inertia of a uniform circular disc about its axis of symmetry and the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone.] **[8]**

A frustum is made by taking a uniform cone of base radius 0.1 m and height 0.2 m and removing a cone of height 0.1 m and base radius 0.05 m as shown in Fig. 4.2. The mass of the frustum is 2.8 kg.

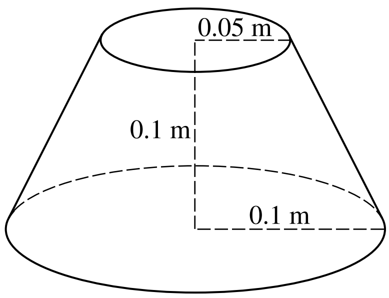

Fig. 4.2

The frustum can rotate freely about its axis of symmetry which is fixed and vertical.

(ii) Show that the moment of inertia of the frustum about its axis of symmetry is $0.0093\,\text{kg m}^2$. **[4]**

The frustum is accelerated from rest for t seconds by a couple of magnitude 0.05 N m about its axis of symmetry, until it is rotating at $10\,\text{rad s}^{-1}$.

(iii) Calculate t. **[4]**

(iv) Find the position of G, the centre of mass of the frustum. **[3]**

The frustum (rotating at $10\,\text{rad s}^{-1}$) then receives an impulse tangential to the circumference of the larger circular face. This reduces its angular speed to $5\,\text{rad s}^{-1}$.

(v) To reduce its angular speed further, a parallel impulse of the same magnitude is now applied tangentially in the horizontal plane through G at the curved surface of the frustum. Calculate the resulting angular speed. **[5]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4764**

Mechanics 4

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet
  (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Thursday 16 June 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (24 marks)

1 A raindrop of mass m falls vertically from rest under gravity. Initially the mass of the raindrop is m_0. As it falls it loses mass by evaporation at a rate λm, where λ is a constant. Its motion is modelled by assuming that the evaporation produces no resultant force on the raindrop. The velocity of the raindrop is v at time t. The forces on the raindrop are its weight and a resistance force of magnitude kmv, where k is a constant.

(i) Find m in terms of m_0, λ and t. **[2]**

(ii) Write down the equation of motion of the raindrop. Solve this differential equation and hence show that $v = \frac{g}{\lambda - k}\left(e^{(\lambda - k)t} - 1\right)$. **[8]**

(iii) Find the velocity of the raindrop when it has lost half of its initial mass. **[2]**

2 A small ring of mass m can slide freely along a fixed smooth horizontal rod. A light elastic string of natural length a and stiffness k has one end attached to a point A on the rod and the other end attached to the ring. An identical elastic string has one end attached to the ring and the other end attached to a point B which is a distance a vertically above the rod and a horizontal distance $2a$ from the point A. The displacement of the ring from the vertical line through B is x, as shown in Fig. 2.

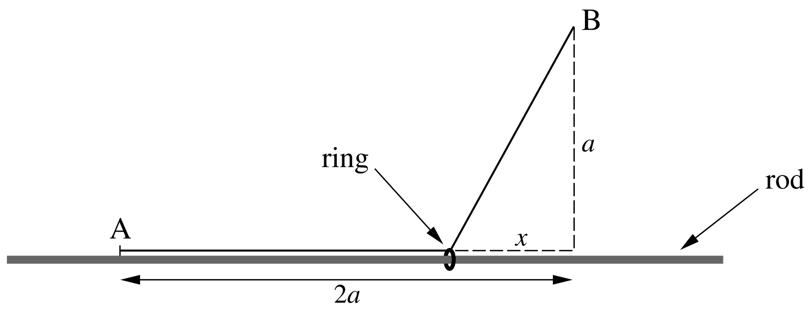

Fig. 2

(i) Find an expression for V, the potential energy of the system when $0 < x < a$, and show that

$$\frac{dV}{dx} = 2kx - ka - \frac{kax}{\sqrt{a^2 + x^2}}.$$ **[5]**

(ii) Show that $\frac{d^2V}{dx^2}$ is always positive. **[4]**

(iii) Show that there is a position of equilibrium with $\frac{1}{2}a < x < a$. State, with a reason, whether it is stable or unstable. **[3]**

Section B (48 marks)

3 A car of mass 800 kg moves horizontally in a straight line with speed $v\,\mathrm{m\,s^{-1}}$ at time t seconds. While $v \leqslant 20$, the power developed by the engine is $8v^4$ W. The total resistance force on the car is of magnitude $8v^2$ N. Initially $v = 2$ and the car is at a point O. At time t s the displacement from O is x m.

(i) Find v in terms of x and show that when $v = 20$, $x = 100\ln 1.9$. **[10]**

(ii) Find the relationship between t and x, and show that when $v = 20$, $t \approx 19.2$. **[6]**

The driving force is removed at the instant when v reaches 20.

(iii) For the subsequent motion, find v in terms of t. Calculate t when $v = 2$. **[8]**

4 In this question you may assume without proof the standard results in *Examination Formulae and Tables (MF2)* for

- the moment of inertia of a disc about an axis through its centre perpendicular to the disc,
- the position of the centre of mass of a solid uniform cone.

Fig. 4 shows a uniform cone of radius a and height $2a$, with its axis of symmetry on the x-axis and its vertex at the origin. A thin slice through the cone parallel to the base is at a distance x from the vertex.

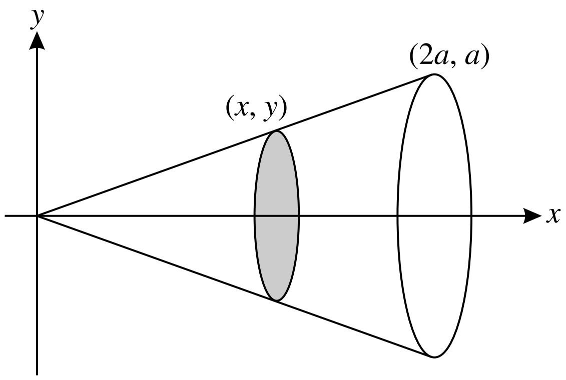

Fig. 4

The slice is taken to be a thin uniform disc of mass m.

(i) Write down the moment of inertia of the disc about the x-axis. Hence show that the moment of inertia of the disc about the y-axis is $\frac{17}{16}mx^2$. **[6]**

(ii) Hence show by integration that the moment of inertia of the cone about the y-axis is $\frac{51}{20}Ma^2$, where M is the mass of the cone. [You may assume without proof the formula for the volume of a cone.] **[8]**

The cone is now suspended so that it can rotate freely about a fixed, horizontal axis through its vertex. The axis of symmetry of the cone moves in a vertical plane perpendicular to the axis of rotation. The cone is released from rest when its axis of symmetry is at an acute angle α to the downward vertical. At time t, the angle the axis of symmetry makes with the downward vertical is θ.

(iii) Use an energy method to show that $\dot{\theta}^2 = \dfrac{20g}{17a}(\cos\theta - \cos\alpha)$. **[5]**

(iv) Hence, or otherwise, show that if α is small the cone performs approximate simple harmonic motion and find the period. **[5]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

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RECOGNISING ACHIEVEMENT

Friday 1 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4764 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4764
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4715820612

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
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- Answer **all** the questions.
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- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
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- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Section A (24 marks)

1 A rocket in deep space has initial mass m_0 and is moving in a straight line at speed v_0. It fires its engine in the direction opposite to the motion in order to increase its speed. The propulsion system ejects matter at a constant mass rate k with constant speed u relative to the rocket. At time t after the engines are fired, the speed of the rocket is v.

(i) Show that while mass is being ejected from the rocket, $(m_0 - kt)\dfrac{\mathrm{d}v}{\mathrm{d}t} = uk$. **[6]**

(ii) Hence find an expression for v at time t. **[5]**

2 A light elastic string AB has stiffness k. The end A is attached to a fixed point and a particle of mass m is attached at the end B. With the string vertical, the particle is released from rest from a point at a distance a below its equilibrium position. At time t, the displacement of the particle below the equilibrium position is x and the velocity of the particle is v.

(i) Show that

$$mv\frac{\mathrm{d}v}{\mathrm{d}x} = -kx.$$ **[4]**

(ii) Show that, while the particle is moving upwards and the string is taut,

$$v = -\sqrt{\frac{k}{m}(a^2 - x^2)}.$$ **[5]**

(iii) Hence use integration to find an expression for x at time t while the particle is moving upwards and the string is taut. **[4]**

Section B (48 marks)

3 A uniform rigid rod AB of length $2a$ and mass m is smoothly hinged to a fixed point at A so that it can rotate freely in a vertical plane. A light elastic string of modulus λ and natural length a connects the midpoint of AB to a fixed point C which is vertically above A with AC = a. The rod makes an angle 2θ with the upward vertical, where $\frac{1}{3}\pi \leqslant 2\theta \leqslant \pi$. This is shown in Fig. 3.

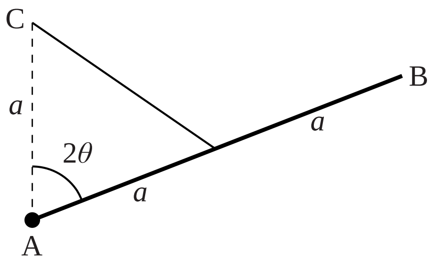

Fig. 3

(i) Find the potential energy, V, of the system relative to A in terms of m, λ, a and θ. Show that

$$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 2a\cos\theta\,(2\lambda\sin\theta - 2mg\sin\theta - \lambda). \quad (*)$$ **[7]**

Assume now that the system is set up so that the result (*) continues to hold when $\pi < 2\theta \leqslant \frac{5}{3}\pi$.

(ii) In the case $\lambda < 2mg$, show that there is a stable position of equilibrium at $\theta = \frac{1}{2}\pi$. Show that there are no other positions of equilibrium in this case. **[9]**

(iii) In the case $\lambda > 2mg$, find the positions of equilibrium for $\frac{1}{3}\pi \leqslant 2\theta \leqslant \frac{5}{3}\pi$ and determine for each whether the equilibrium is stable or unstable, justifying your conclusions. **[7]**

4 **(i)** Show by integration that the moment of inertia of a uniform circular lamina of radius a and mass m about an axis perpendicular to the plane of the lamina and through its centre is $\frac{1}{2}ma^2$. **[6]**

A closed hollow cylinder has its curved surface and both ends made from the same uniform material. It has mass M, radius a and height h.

(ii) Show that the moment of inertia of the cylinder about its axis of symmetry is $\frac{1}{2}Ma^2\left(\frac{a+2h}{a+h}\right)$. **[6]**

For the rest of this question take the cylinder to have mass 8 kg, radius 0.5 m and height 0.3 m.

The cylinder is at rest and can rotate freely about its axis of symmetry. It is given a tangential impulse of magnitude 55 N s at a point on its curved surface. The impulse is perpendicular to the axis.

(iii) Find the angular speed of the cylinder after the impulse. **[3]**

A resistive couple is now applied to the cylinder for 5 seconds. The magnitude of the couple is $2\dot{\theta}^2$ N m, where $\dot{\theta}$ is the angular speed of the cylinder in rad s^{-1}.

(iv) Formulate a differential equation for $\dot{\theta}$ and hence find the angular speed of the cylinder at the end of the 5 seconds. **[7]**

The cylinder is now brought to rest by a constant couple of magnitude 0.03 N m.

(v) Calculate the time it takes from when this couple is applied for the cylinder to come to rest. **[3]**

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RECOGNISING ACHIEVEMENT

Monday 10 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4764/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4764/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4715820613

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

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INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Section A (24 marks)

1 An empty railway truck of mass m_0 is moving along a straight horizontal track at speed v_0. The point P is at the front of the truck. The horizontal forces on the truck are negligible. As P passes a fixed point O, sand starts to fall vertically into the truck at a constant mass rate k. At time t after P passes O the speed of the truck is v and OP $= x$.

(i) Find an expression for v in terms of m_0, v_0, k and t, and show that $x = \dfrac{m_0 v_0}{k} \ln\left(1 + \dfrac{kt}{m_0}\right)$. **[9]**

(ii) Find the speed of the truck and the distance OP when the mass of sand in the truck is $2m_0$. **[2]**

2 A uniform rod AB of length 0.5 m and mass 0.5 kg is freely hinged at A so that it can rotate in a vertical plane. Attached at B are two identical light elastic strings BC and BD each of natural length 0.5 m and stiffness 2 N m^{-1}. The ends C and D are fixed at the same horizontal level as A and with AC = CD = 0.5 m. The system is shown in Fig. 2.1 with the angle BAC $= \theta$. You may assume that $\frac{1}{3}\pi \leqslant \theta \leqslant \frac{5}{3}\pi$ so that both strings are taut.

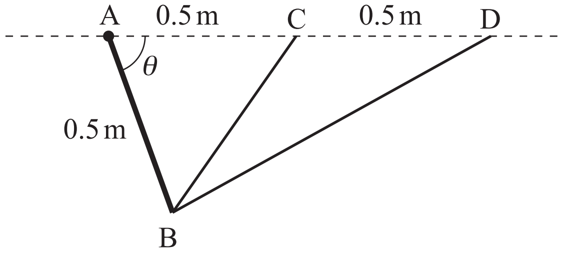

Fig. 2.1

(i) Show that the length of BC in metres is $\sin\frac{1}{2}\theta$. **[1]**

(ii) Find the potential energy, V J, of the system relative to AD in terms of θ. Hence show that

$$\frac{dV}{d\theta} = 1.5\sin\theta - 1.225\cos\theta - \frac{0.5\sin\theta}{\sqrt{1.25 - \cos\theta}} - 0.5\cos\frac{1}{2}\theta.$$

[8]

(iii) Fig. 2.2 shows a graph of the function $f(\theta) = 1.5\sin\theta - 1.225\cos\theta - \dfrac{0.5\sin\theta}{\sqrt{1.25 - \cos\theta}} - 0.5\cos\frac{1}{2}\theta$ for $0 \leqslant \theta \leqslant 2\pi$.

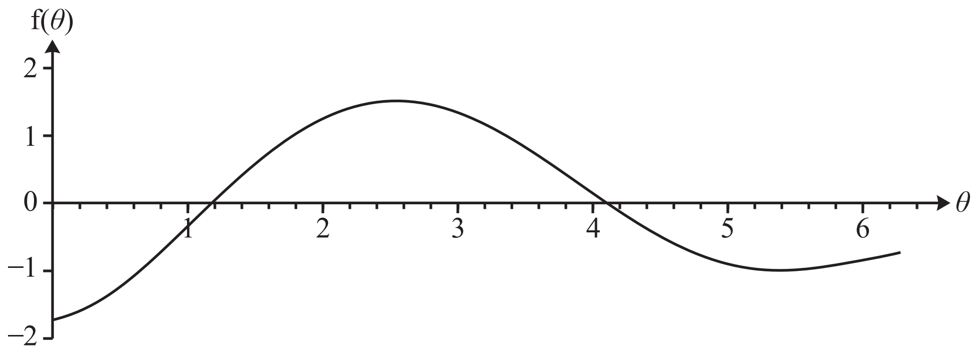

Fig. 2.2

Use the graph both to estimate, correct to 1 decimal place, the values of θ for which the system is in equilibrium and also to determine their stability. **[4]**

Section B (48 marks)

3 A model car of mass 2 kg moves from rest along a horizontal straight path. After time t s, the velocity of the car is $v\,\mathrm{m\,s^{-1}}$. The power, P W, developed by the engine is initially modelled by $P = 2v^3 + 4v$. The car is subject to a resistance force of magnitude $6v$ N.

(i) Show that $\dfrac{dv}{dt} = (1 - v)(2 - v)$ and hence show that $t = \ln\dfrac{2 - v}{2(1 - v)}$. **[10]**

(ii) Hence express v in terms of t. **[2]**

Once the power reaches 4.224 W it remains at this constant value with the resistance force still acting.

(iii) Verify that the power of 4.224 W is reached when $v = 0.8$ and calculate the value of t at this instant. **[2]**

(iv) Find v in terms of t for the motion at constant power. Deduce the limiting value of v as $t \to \infty$. **[10]**

4 A uniform lamina of mass m is in the shape of a sector of a circle of radius a and angle $\frac{1}{3}\pi$. It can rotate freely in a vertical plane about a horizontal axis perpendicular to the lamina through its vertex O.

(i) Show by integration that the moment of inertia of the lamina about the axis is $\frac{1}{2}ma^2$. **[6]**

(ii) State the distance of the centre of mass of the lamina from the axis. **[1]**

The lamina is released from rest when one of the straight edges is horizontal as shown in Fig. 4.1. After time t, the line of symmetry of the lamina makes an angle θ with the downward vertical.

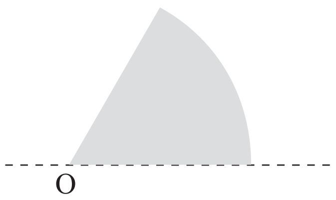

Fig. 4.1

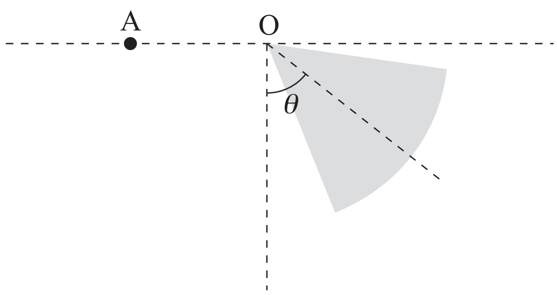

Fig. 4.2

(iii) Show that $\dot{\theta}^2 = \dfrac{4g}{\pi a}(2\cos\theta + 1)$. **[4]**

(iv) Find the greatest speed attained by any point on the lamina. **[4]**

(v) Find an expression for $\ddot{\theta}$ in terms of θ, a and g. **[2]**

The lamina strikes a fixed peg at A where $AO = \frac{3}{4}a$ and is horizontal, as shown in Fig. 4.2. The collision reverses the direction of motion of the lamina and halves its angular speed.

(vi) Find the magnitude of the impulse that the peg gives to the lamina. **[4]**

(vii) Determine the maximum value of θ in the subsequent motion. **[3]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.